Review: Calculus I

Section 4.9: Antiderivatives

Objective: In this lesson, you learn

☐ how to find antiderivatives of functions.

I. Antiderivatives

Definition

A function F is called an **antiderivative** of f on an interval I if

$$F'(x) = f(x)$$

for all x in I.

Remark: If two functions have identical derivatives on an interval, then by the Mean Value Theorem, they must differ by a constant.

Thus, if F and G are any two antiderivatives of f, then $F'(x) = f(x) = G'(x)$. So

$$G(x) = F(x) + C.$$

Example 1: Find an antiderivative of f of the following functions

a. $f(x) = x^2$

b. $f(x) = \dfrac{1}{x}$

Theorem

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

II. Antidifferentiation

The following table lists some of Antiderivative Formulas. (Assume $F' = f$ and $G' = g$.)

Some of Antiderivative/derivative Formulas

Antiderivative	**Function**	**Derivative**
$F(x) + C$	$f(x)$	$f'(x)$
$F(x) \pm G(x) + C$	$f(x) \pm g(x)$	$f'(x) \pm g'(x)$
$kx + C$	$k,\ k-$constant	0
$\dfrac{x^{n+1}}{n+1} + C$	$x^n \ (n \neq -1)$	nx^{n-1}
$\dfrac{2x^{3/2}}{3} + C$	$\sqrt{x}$	$\dfrac{1}{2\sqrt{x}}$
$\dfrac{b^x}{\ln b} + C$	b^x	$b^x \ln b$
$e^x + C$	e^x	e^x
$\ln\lvert x\rvert + C$	$\dfrac{1}{x}$	$\dfrac{-1}{x^2}$
$-\cos x + C$	$\sin x$	$\cos x$
$\sin x + C$	$\cos x$	$-\sin x$
$\tan x + C$	$\sec^2 x$	$2\sec^2 x \tan x$
$-\cot x + C$	$\csc^2 x$	$-2\csc^2 x \cot x$
$\sec x + C$	$\sec x \tan x$	$\sec x\left(\sec^2 x + \tan^2 x\right)$
$\csc x + C$	$-\csc x \cot x$	$\csc x\left(\csc^2 x + \cot^2 x\right)$
$\tan^{-1} x + C$	$\dfrac{1}{1+x^2}$	$\dfrac{-2x}{(1+x^2)^2}$
$\sin^{-1} x + C$	$\dfrac{1}{\sqrt{1-x^2}}$	$\dfrac{x}{(1-x^2)^{(5/2)}}$

Example 2: Find all anti-derivative of

$$f(x) = x^3 + 3\sqrt{x} + \frac{4}{x} + 2$$

Example 3:

a. Find f if $f'(t) = 2\cos(t) + 3e^t$

b. Which of the functions in part (a) satisfies $f(0) = 0$?